Question Style in HKDSE Paper 1

Candidates should pay close attention to the following question types in the examination.

Short questions in Section B

There may be some short questions in Section B which carry less marks. Since such questions may not have parts to guide candidates, they are set to differentiate high ability candidates from others. Candidates who aim to obtain 5** in the HKDSE should strive to master this type of question.

Examples in this book:

16. There are 8 boys and 7 girls in a group of students. 4 boys and 3 girls are randomly selected to line up in a queue.
   (a) Find the number of ways of arranging the queue. (2 marks)
   (b) Find the probability that the 4 boys are next to each other in the queue. (3 marks)

Questions on real-life situations

These questions require candidates to apply mathematical knowledge and techniques to real-life situations. Some of them may relate to the topics on ‘Further Applications’ in the syllabus.

Examples in this book:

18. The graph in Figure 6 shows the linear relation between $\log_{10} x$ and $\log_{10} y$. The slope and the intercept on the horizontal axis of the graph are $\frac{1}{2}$ and $-4$ respectively. Express the relation between $x$ and $y$ in the form $y = ax^b$, where $a$ and $b$ are constants.

   (3 marks)

18. The data below show the heights of the members in a basketball team:

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>2</td>
</tr>
<tr>
<td>180</td>
<td>3</td>
</tr>
<tr>
<td>185</td>
<td>2</td>
</tr>
<tr>
<td>191</td>
<td>1</td>
</tr>
<tr>
<td>178</td>
<td>2</td>
</tr>
<tr>
<td>183</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Find the range and the inter-quartile range of the above data. (2 marks)
(b) Later, the two shortest members leave the team, while a new member joins the team. Write down the maximum possible value of the new inter-quartile range of the heights of the members. (2 marks)
(c) It is known that the range of the heights of the members remains unchanged. Find all possible values of the new mean height of the members. (4 marks)
D  Question Style in HKDSE Paper 2

According to the latest HKDSE papers, more questions are set using variables and notations rather than numerical values. Candidates should prepare to handle such questions.

About Number and Algebra

Examples in this book:

8. The figure shows the graph of \( y = ax^2 + bx + c \), where \( b < a \). Given that the axis of symmetry of the graph cuts the \( x \)-axis at \( (1, 0) \), which of the following are true?

- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II and III

About Measures, Shape and Space

Examples in this book:

26. In the figure, the straight lines \( L_1: x + 2y + 3 = 0 \) and \( L_2: x = 2y + 4 = 0 \) intersect at a point on the \( y \)-axis. Which of the following are true?

- A. \( a < b \)
- B. \( b < a \)
- C. \( a = b \)
- D. I, II and III

About Data Handling

Examples in this book:

30. Let \( A \) be a group of numbers \( \{a, b, c, d, e\} \) and \( B \) be another group of numbers \( \{2a, b, 3a, d, e\} \). Which of the following must be true?

- A. \( a < e \)
- B. \( b < c \)
- C. \( c < d \)
- D. I, II and III
SECTION B (35 marks)

15. The graph in Figure 6 shows the linear relation between $\log_9 x$ and $\log_{27} y$. The slope and the intercept on the horizontal axis of the graph are $\frac{1}{4}$ and $-4$ respectively. Express the relation between $x$ and $y$ in the form $y = Ax^k$, where $A$ and $k$ are constants. (3 marks)
14. The following stem-and-leaf diagram shows the distribution of the ages of the members of a tennis club.

<table>
<thead>
<tr>
<th>Stem (tens)</th>
<th>Leaf (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 8 8 9 9</td>
</tr>
<tr>
<td>2</td>
<td>0 0 1 3 3 b 9</td>
</tr>
<tr>
<td>3</td>
<td>1 2 3 5 9 9</td>
</tr>
<tr>
<td>4</td>
<td>0 8 8 a</td>
</tr>
</tbody>
</table>

It is known that the median and the range of the distribution are 27 and 31 respectively.

(a) Find $a$ and $b$. (3 marks)

(b) A few days later, both the oldest and the youngest members leave the club, while Dennis and his father join the club. It is found that the changes in members have not changed the mode and the median of the distribution.

(i) Find the age of Dennis’s father.

(ii) Is it possible that Dennis is 25 years old? Explain your answer.

(iii) The ages of the members of a badminton club are shown as follow:

17, 22, 22, 32, 35, 36, 39, 40

One member from the badminton club and one member from the tennis club are randomly selected to be the club representatives. Find the probability that the sum of the ages of the two representatives exceeds 65. (6 marks)
28. A box contains seven balls marked with the numbers \(-3, -2, -1, 1, 2, 3\) and 4 respectively. If two balls are drawn randomly from the box at the same time, find the probability that the product of the numbers on the balls drawn is positive.

\[
\begin{align*}
A. & \quad \frac{5}{42} \\
B. & \quad \frac{1}{6} \\
C. & \quad \frac{2}{7} \\
D. & \quad \frac{3}{7}
\end{align*}
\]

29. The stem-and-leaf diagram below shows the distribution of the numbers of books read by a group of students in the first school term.

<table>
<thead>
<tr>
<th>Stem (tens)</th>
<th>Leaf (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>3 8 9</td>
</tr>
<tr>
<td>2</td>
<td>0 h 4 5 6 8 8</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 0 1 k 7 7 7 8 9</td>
</tr>
</tbody>
</table>

If the inter-quartile range of the above distribution is at least 14, which of the following must be true?

I. \(0 \leq h \leq 4\)

II. \(4 \leq k \leq 7\)

III. \(1 \leq k - h \leq 3\)

A. I only

B. II only

C. I and III only

D. II and III only
21. In the figure, \(O\) is the centre of the circle \(ABCD\). If \(\angle AOD = 120^\circ\) and \(\angle ODC = 70^\circ\), find \(\angle ABC\).

A. 100°  
B. 110°  
C. 120°  
D. 140°

22. In the figure, \(O\) is the centre of the circle \(ABCDEF\). \(\triangle PQR\) intersects the circle at \(A, B, C, D, E\) and \(F\). If \(\angle POQ = 118^\circ\) and \(AB = CD = EF\), then \(\angle PRQ =

A. 28°.  
B. 56°.  
C. 59°.  
D. 62°.

23. If an exterior angle of a regular \(n\)-sided polygon is smaller than an interior angle by 90°, which of the following is/are true?

I. The value of \(n\) is 8.

II. The number of the diagonals of the polygon is 8.

III. The number of axes of reflectional symmetry of the polygon is 8.

A. I only  
B. II only  
C. I and III only  
D. II and III only
HKDSE Mathematics
Multiple Choice Questions

C M Yeung  K Y Lee
W K Ching  K K Hui

Solution Guide CD-ROM Included

- HKDSE and HKCEE trend analysis
- Useful techniques in tackling MCQs
- Intensive Revision Notes
- Quick Drill for checking basic skills
- Sectional Exercises and Quizzes
- Mock Tests (Paper 2)
Useful Techniques in Tackling MCQ

Useful Techniques in Tackling MCQ

In HKDSE Mathematics Paper 2, candidates need to answer 45 multiple choices questions within 1 hour 15 minutes. On average, candidates can only spend about 1.5 minutes on each question. Apart from direct computation and deductive reasoning, the following six common techniques may help candidates to tackle some of the multiple choice questions quickly.

1. Checking for Questions WITHOUT Graphs
2. Checking for Questions WITH Graphs
3. Observing a Particular Case
4. Substituting a Particular Value
5. Elimination
6. Construction

Technique 1: Checking for Questions WITHOUT Graphs

Example

If the 2nd term and the 5th term of an arithmetic sequence are 13 and 1 respectively, find the 4th term of the sequence.
A. 17
B. 9
C. 5
D. −4

(Reference: HKDSE Sample Paper 2009 Q36)

Solution

(by Checking)
The arithmetic sequence is:
_____, 13, _____, _____, 1
Option C: The 4th term is 5. Then, the common difference = 1 − 5 = −4
The following arithmetic sequence obtained satisfies all the given conditions:
17, 13, 9, 5, 1
∴ The answer is C.

(by Direct Computation)
Let a and d be the first term and the common difference of the sequence.
∴ 2nd term = 13
∴ a + d = 13 ......(1)
∴ 5th term = 1
∴ a + 4d = 1 ......(2)
(2) − (1): 3d = −12
∴ d = −4
From (1), a + d = 13
(a + d) + 2d = 13 + 2d
a + 3d = 13 + 2(−4)
= 5
∴ The 4th term is 5.
∴ The answer is C.
8.3 Quadratic Equations

A Solving quadratic equations by the factor method
If \((ax + b)(cx - d) = 0\), where \(a, c \neq 0\), then
\[x = \frac{b}{a} \text{ or } x = \frac{d}{c}.
\]

B Forming quadratic equations with given real roots
If \(\alpha\) and \(\beta\) are the roots of a quadratic equation in \(x\), then the equation can be written as:
\[(x - \alpha)(x - \beta) = 0\]

Common Mistakes
Students may mistakenly write \((x + \alpha)(x + \beta) = 0\) when forming a quadratic equation with two given roots \(\alpha\) and \(\beta\).

C Solving quadratic equations by the quadratic formula
If \(ax^2 + bx + c = 0\), where \(a \neq 0\), then
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Exam Tips
Some MC questions require students to find roots of a quadratic equation. If the given options involve surds, that means the equation may not be easily solved by the factor method. In such case, we should solve the equation using the quadratic formula.

D Solving quadratic equations by the graphical method
If the \(x\)-intercepts of the graph of \(y = ax^{2} + bx + c\) are \(r\) and \(s\), then the roots of the quadratic equation \(ax^{2} + bx + c = 0\) are also \(r\) and \(s\).

Quick Drill 3
If \(4(x + 5)(2x - 7) = 0\), then \(x = \)
A. \(5 \text{ or } \frac{7}{2}\)
B. \(4 \text{ or } \frac{7}{2}\)
C. \(-5 \text{ or } \frac{7}{2}\)
D. \(4 \text{ or } \frac{7}{2}\)

Quick Drill 4
Form a quadratic equation in \(x\) whose roots are \(-4\) and \(\frac{1}{2}\).
A. \((x - 4)(5x + 2) = 0\)
B. \((x + 4)(5x - 2) = 0\)
C. \((x - 4)(2x + 5) = 0\)
D. \((x + 4)(2x - 5) = 0\)

Quick Drill 5
Solve \(x^{2} + 3x - 1 = 0\).
A. \(x = \frac{-3 \pm \sqrt{5}}{2}\)
B. \(x = \frac{3 \pm \sqrt{5}}{2}\)
C. \(x = \frac{-3 \pm \sqrt{13}}{2}\)
D. \(x = \frac{3 \pm \sqrt{13}}{2}\)

Quick Drill 6
Which of the following equations may be represented by the given graph?
A. \(y = (x - 1)(x + 3)\)
B. \(y = (x - 1)(x - 3)\)
C. \(y = (x + 1)(x + 3)\)
D. \(y = (x + 1)(x - 3)\)
Basic Properties of Circles

HKDSE and HKCEE Mathematics Paper 2 Questions Distribution

| 17.1 | Chords of a Circle |
| A | Perpendiculars to chords |
| B | Distances between chords and centre |

| 17.2 | Angles in a Circle |
| A | Angles at the centre and angles at the circumference |
| B | Angles in the same segment |

| 17.3 | Relationships among Arcs, Chords and Angles |
| A | Equal arcs, equal chords and equal angles |
| B | Arcs proportional to angles at the centre |

| 17.4 | Cyclic Quadrilaterals |

| 17.5 | Tests for Concyclic Points |

Past Public Exam Questions

| 10, Q49 |

| 05, Q25; 06, Q46; 09, Q48, Q49; Sample Paper, Q21 |

| 01, Q32; 02, Q29; 03, Q25; 04, Q50; 05, Q51 |

| 03, Q50, Q51; 07, Q48; 08, Q50; Sample Paper, Q22 |

**Exercise**

Part I Sectional Exercise

17.1 Chords of a Circle

1. In the figure, $AMB$ and $BNC$ are straight lines. $AB = 10$ cm and $BC = 16$ cm. Find the area of rectangle $OMBN$.

   A. $30$ cm$^2$
   B. $40$ cm$^2$
   C. $80$ cm$^2$
   D. $160$ cm$^2$

2. In the figure, $AMB$ and $COM$ are straight lines. If $AM = MB = 8$ cm and $OM = 6$ cm, find $CM$.

   A. $10$ cm
   B. $12$ cm
   C. $15$ cm
   D. $16$ cm

3. In the figure, $AOB$ is a diameter of the circle and $DMC$ is a straight line. If $AB = 12$ cm and $DC = 10$ cm, then $OM =$

   A. $3$ cm.
   B. $\sqrt{11}$ cm.
   C. $\sqrt{15}$ cm.
   D. $4$ cm.

4. In the figure, $AMB$ and $OMC$ are straight lines. If $AM = MB = 12$ cm and $OC = 15$ cm, find $MC$.

   A. $6$ cm
   B. $7.5$ cm
   C. $9$ cm
   D. $10.5$ cm
40. The figure shows \( \triangle ABC \). FD intersects BE at G. If \( FD \perp BC \) and \( BE \perp AC \), which of the following is/are true?

I. A, F, G and E are concyclic.
II. E, G, D and C are concyclic.
III. A, E, D and B are concyclic.

A. I only
B. II only
C. I and III only
D. II and III only

43. In the figure, \( \triangle ABC \) is a straight line. If \( OB = 3 \) and \( OA = 4 \), find \( BC \).

A. 1.4
B. 1.6
C. 1.8
D. 2

44. In the figure, \( \angle AOB = \angle BOC \) and \( OA \parallel CB \). Which of the following is/are true?

I. \( AB = BC \)
II. \( BC = OB \)
III. \( OC \parallel AB \)

A. I only
B. II only
C. I and II only
D. I, II and III

45. In the figure, \( M \) and \( N \) are the mid-points of the chords \( AB \) and \( CD \) respectively. ONM is a straight line. If \( AB : CD = 1 : 2 \), which of the following is/are true?

I. \( AB : CD = 1 : 2 \)
II. \( ON : OM = 1 : 2 \)
III. \( \cos x : \cos y = 1 : 2 \)

A. I only
B. III only
C. I and III only
D. II and III only
Measures, Shape and Space

**Quiz**

Time allowed: 30 min

1. In the figure, $AB = 20$ and $OA = 26$. Find the shortest distance from $O$ to chord $AB$.
   - A. 6
   - B. $2\sqrt{69}$
   - C. 24
   - D. $2\sqrt{194}$

2. The figure shows two concentric circles with common centre $O$. $ABMCD$ is a straight line with $BM = MC = 6$. $OA = 12$ and $OC = 10$. Find $AD$.
   - A. $8\sqrt{5}$
   - B. $10\sqrt{5}$
   - C. $5\sqrt{10}$
   - D. $6\sqrt{10}$

3. In the figure, $OA \parallel CB$. Find $\angle BAC$.
   - A. $15^\circ$
   - B. $20^\circ$
   - C. $35^\circ$
   - D. $75^\circ$

4. In the figure, $AB$ is a diameter of the circle. $DEB$ and $CEO$ are straight lines. Find $x$.
   - A. $25^\circ$
   - B. $26^\circ$
   - C. $27^\circ$
   - D. $28^\circ$
There are 30 questions in Section A and 15 questions in Section B. The diagrams in this paper are not necessarily drawn to scale. Choose the best answer for each question.

Section A

1. \((-2^{32})^3 \left(\frac{1}{2}\right)^{366}\) =
   A. -1 B. 0 C. 1 D. \(\frac{1}{2^{366}}\)

2. If \(3x - 4y = 5xy\), then \(y =\)
   A. \(\frac{3x}{5x + 4}\) B. \(\frac{3x}{5x - 4}\)
   C. \(\frac{5x + 4}{3x}\) D. \(\frac{5x - 4}{3x}\)

3. \(ab - bc - ad + cd =\)
   A. \((a - c)(d - b)\) B. \((a - c)(b - d)\)
   C. \((a + c)(d - b)\) D. \((a + c)(b - d)\)

4. Which of the following is an identity/are identities?
   I. \(4x^2 - 9 = 0\)
   II. \(4x^2 - 9 = (2x - 3)^2\)
   III. \(4x^2 - 9 = (2x - 3)(2x + 3)\)
   A. II only B. III only C. I and II only D. II and III only

5. Let \(f(x) = -2x^2 + x - 1\). If \(f(a) = f(-a)\), find the value(s) of \(a\).
   A. 0 B. 1 C. 0 or \(\frac{1}{2}\) D. 0 or \(-\frac{1}{2}\)

6. Let \(f(x) = -2x^3 + x^2 - x + k\). If \(x + 1\) is a factor of \(f(x)\), find the remainder when \(f(x)\) is divided by \(x - 2\).
   A. -18 B. -12 C. 18 D. 24

7. Which of the following about the graph of quadratic function \(y = -1 - 2(x - 3)^2\) are true?
   I. The graph has no x-intercepts.
   II. The coordinates of the vertex are \((-3, -1)\).
   III. The axis of symmetry of the graph is \(x - 3 = 0\).
   A. I and II only B. I and III only C. II and III only D. I, II and III

8. Find the range of values of \(k\) such that the quadratic equation \(x^2 - 4x + k = 2\) has real roots.
   A. \(k \leq 4\) B. \(k \geq 4\)
   C. \(k \leq 6\) D. \(k \geq 6\)

9. If \(a\) and \(b\) are real numbers such that \(ab < 0\), which of the following must be true?
   I. \(\frac{a}{b} < 0\)
   II. \(a + b < 0\)
   III. \(a^2 - b^2 < 0\)
   A. I only B. II only C. I and III only D. II and III only

10. If \(x\) is a positive integer satisfying the inequality 
    \(3x^2 - 10x - 8 < 0\), then the smallest possible value of \(x\) is
Section B

31. \( \frac{x + 2}{x^2 - 5x + 6} - \frac{x - 3}{x^2 - x - 2} = \)
   A. \( \frac{3x + 11}{(x - 3)(x - 2)(x + 1)} \)
   B. \( \frac{3x - 7}{(x - 3)(x - 2)(x + 1)} \)
   C. \( \frac{-3x + 11}{(x - 3)(x - 2)(x + 1)} \)
   D. \( \frac{9x - 7}{(x - 3)(x - 2)(x + 1)} \)

32. The graph in the figure shows the linear relation between \( x \) and \( \log_4 y \). If \( y = ab^x \), find the values of \( a \) and \( b \).

33. Which of the following have the same value as 2012\(_{10}\)?
   I. \( 2 \times 10^4 + 0 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 \)
   II. 7DC\(_{16}\)
   III. 1111011100\(_{2}\)
   A. I and II only
   B. I and III only
   C. II and III only
   D. I, II and III

34. \( i^{2012} = \)
   A. \( i \)
   B. \( -i \)
   C. 1
   D. \( -1 \)

35. Which region in the figure represents the solutions of \( \begin{cases} x \geq y \\ x + y \leq 12 \\ 3x + y \geq 12 \end{cases} \)?
   A. Region I
   B. Region II
   C. Region III
   D. Region IV

36. Let \( a, b \) and \( c \) be positive integers. If \( a, b, c \) is an arithmetic sequence, which of the following must be true?
   I. \( a^2, ab, ac \) is an arithmetic sequence.
   II. \( 2^a, 2^b, 2^c \) is a geometric sequence.
   III. \( a^2, b^2, c^2 \) is not an arithmetic sequence.
   A. I and II only
   B. I and III only
   C. II and III only
   D. I, II and III

37. The sum of all the negative terms in the geometric sequence \( \frac{-2}{3}, \frac{1}{2}, -\frac{3}{8}, \ldots \) is
   A. \( \frac{8}{21} \)
   B. \( \frac{32}{21} \)
   C. \( \frac{8}{3} \)
   D. \( \frac{32}{75} \)